國立中正大學九十八學年度學士班二年級轉學生招生考試試題

學系別：數學系、地球與環境科學系、物理學系、化學暨生物化學系、資訊工程學系、經濟學系、電機工程學系、機械工程學系、通訊工程學系　　科目：微積分

第 1 節　　第 1 頁，共 1 頁

★ Write down your answers without calculations in problems 1 ~ 5.

1. Evaluate the following limits:

(a). $\lim_{x\to 1} \dfrac{(1-\sqrt{x})(1-\sqrt[3]{x})(1-\sqrt[4]{x})}{(1-x)^3} =$ ______. (5 pts.)

(b). $\lim_{x\to 0} (e^{3x}+2x)^{\frac{1}{x}} =$ ______. (5 pts.)

(c). $\lim_{n\to\infty} \left(\dfrac{1}{\sqrt{n^2+1}}+\dfrac{1}{\sqrt{n^2+2}}+\cdots+\dfrac{1}{\sqrt{n^2+n}}\right) =$ ______. (5 pts.)

(d). $\lim_{n\to\infty} \dfrac{1}{n^{p+1}}(1^p+2^p+\cdots+n^p) =$ ______ where $p > 0$. (5 pts.)

2. Suppose that $f(x) = \begin{cases} ax-b & \text{if } x \leqslant 1 \\ x^3 & \text{if } x > 1 \end{cases}$ is a differentiable function on $\mathbb{R}$. Then

$\lim_{x\to 0^+} \dfrac{1}{x\sqrt{x}}\left(a\tan^{-1}\left(\dfrac{\sqrt{x}}{\sqrt{a}}\right) - b\tan^{-1}\left(\dfrac{\sqrt{x}}{\sqrt{b}}\right)\right) =$ ______. (15 pts.)

3. Let $a_n = \int_0^{\frac{\pi}{2}} (\sin x)^n dx$ where $n \in \mathbb{N}\cup\{0\}$. Then

(a). $\dfrac{a_n}{a_{n-2}} =$ ______ where $n \in \mathbb{N}$ and $n \geqslant 2$. (10 pts.)

(b). $a_n =$ ______ if n is even. (5 pts.)

(c). $a_n =$ ______ if n is odd. (5 pts.)

4. Assume that $f(x) = \sqrt{x^3}$. Define $\Gamma = \{(x, f(x)) \in \mathbb{R}^2 | 0 \leqslant x \leqslant 4\}$ and $R = \{(x,y) \in \mathbb{R}^2 | 0 \leqslant x \leqslant 4, 0 \leqslant y \leqslant f(x)\}$. Then

(a). The arc length of $\Gamma =$ ______. (5 pts.)

(b). The volume of the solid obtained by rotating R about the x−axis is ______. (5 pts.)

5. Suppose that $\int\int_S \left(\sqrt{\left(\dfrac{y}{x}\right)^3} + \sqrt{xy}\right) dxdy = \int\int_T (v^3+u)\dfrac{\partial(x,y)}{\partial(u,v)} dudv = A$,

where $S = \{(x,y) \in \mathbb{R}^2 | 1 \leqslant xy \leqslant 9, 1 \leqslant y/x \leqslant 4\}$. Then

(a). $T =$ ______. (5 pts.)

(b). $A =$ ______. (5 pts.)

6. Show that if $a_0, a_1, \cdots, a_n$ are real numbers satisfying

$$\frac{a_0}{1}+\frac{a_1}{2}+\cdots+\frac{a_n}{n+1} = 0,$$

then the equation $a_0 + a_1x + a_2x^2 + \cdots + a_nx^n = 0$ has at least one real root. (10 pts.)

7. Find extrema of $f(x,y,z) = x^3+y^3+z^3$ subject to $x^2+2y^2+3z^2 = 4$. (15 pts.)