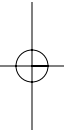
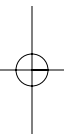
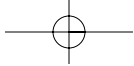


# Advanced Engineering Mathematics





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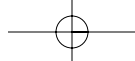
# Advanced Engineering Mathematics

ERWIN KREYSZIG

Professor of Mathematics  
Ohio State University  
Columbus, Ohio




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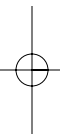
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# P R E F A C E

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## Goal of the Book. Arrangement of Material

This new edition continues the tradition of providing instructors and students with a comprehensive and up-to-date resource for teaching and learning *engineering mathematics*, that is, *applied mathematics* for engineers and physicists, mathematicians and computer scientists, as well as members of other disciplines. A course in elementary calculus is the sole *prerequisite*.

The subject matter is arranged into seven parts A–G:

- A Ordinary Differential Equations (ODEs) (Chaps. 1–6)
- B Linear Algebra. Vector Calculus (Chaps. 7–9)
- C Fourier Analysis. Partial Differential Equations (PDEs) (Chaps. 11–12)
- D Complex Analysis (Chaps. 13–18)
- E Numeric Analysis (Chaps. 19–21)
- F Optimization, Graphs (Chaps. 22–23)
- G Probability, Statistics (Chaps. 24–25).

This is followed by five appendices:

- App. 1 References (ordered by parts)
- App. 2 Answers to Odd-Numbered Problems
- App. 3 Auxiliary Material (see also inside covers)
- App. 4 Additional Proofs
- App. 5 Tables of Functions.

This book has helped to pave the way for the present development of engineering mathematics. By a modern approach to those areas A–G, this new edition will prepare the student for the tasks of the present and of the future. The latter can be predicted to some extent by a judicious look at the present trend. Among other features, this trend shows the appearance of more complex production processes, more extreme physical conditions (in space travel, high-speed communication, etc.), and new tasks in robotics and communication systems (e.g., fiber optics and scan statistics on random graphs) and elsewhere. This requires the refinement of existing methods and the creation of new ones.

It follows that students need solid knowledge of basic principles, methods, and results, and a clear view of what engineering mathematics is all about, and that it requires proficiency in all three phases of problem solving:

- **Modeling**, that is, translating a physical or other problem into a mathematical form, into a mathematical *model*; this can be an algebraic equation, a differential equation, a graph, or some other mathematical expression.
- **Solving** the model by selecting and applying a suitable mathematical method, often requiring numeric work on a computer.
- **Interpreting** the mathematical result in physical or other terms to see what it practically means and implies.

It would make no sense to overload students with all kinds of little things that might be of occasional use. Instead they should recognize that mathematics rests on relatively few basic concepts and involves powerful unifying principles. This should give them a firm grasp on the *interrelations among theory, computing, and* (physical or other) *experimentation*.

## PARTS AND CHAPTERS OF THE BOOK

<b>PART A</b>	
Chaps. 1–6 Ordinary Differential Equations (ODEs)	
Chaps. 1–4 Basic Material	
↓ Chap. 5 Series Solutions	Chap. 6 Laplace Transforms ↓

<b>PART B</b>	
Chaps. 7–10 Linear Algebra. Vector Calculus	
Chap. 7 Matrices, Linear Systems	Chap. 9 Vector Differential Calculus
↓ Chap. 8 Eigenvalue Problems	Chap. 10 Vector Integral Calculus ↓

<b>PART C</b>	
Chaps. 11–12 Fourier Analysis. Partial Differential Equations (PDEs)	
Chap. 11 Fourier Analysis	
↓ Chap. 12 Partial Differential Equations	

<b>PART D</b>	
Chaps. 13–18 Complex Analysis, Potential Theory	
Chaps. 13–17 Basic Material	
↓ Chap. 18 Potential Theory	

<b>PART E</b>		
Chaps. 19–21 Numeric Analysis		
Chap. 19 Numerics in General	Chap. 20 Numeric Linear Algebra	Chap. 21 Numerics for ODEs and PDEs

<b>PART F</b>	
Chaps. 22–23 Optimization, Graphs	
Chap. 22 Linear Programming	Chap. 23 Graphs, Optimization

<b>PART G</b>	
Chaps. 24–25 Probability, Statistics	
Chap. 24 Data Analysis. Probability Theory	
↓ Chap. 25 Mathematical Statistics	

<b>GUIDES AND MANUALS</b>	
Maple Computer Guide Mathematica Computer Guide	
Student Solutions Manual	
Instructor's Manual	

## General Features of the Book Include:

- **Simplicity of examples**, to make the book teachable—why choose complicated examples when simple ones are as instructive or even better?
- **Independence of chapters**, to provide *flexibility* in tailoring courses to special needs.
- **Self-contained presentation**, except for a few clearly marked places where a proof would exceed the level of the book and a reference is given instead.
- **Modern standard notation**, to help students with other courses, *modern* books, and mathematical and engineering journals.

Many sections were rewritten in a more detailed fashion, to make it a *simpler book*. This also resulted in a *better balance between theory and applications*.

## Use of Computers

The presentation is *adaptable to various levels of technology and use of a computer or graphing calculator*: very little or no use, medium use, or intensive use of a graphing calculator or of an unspecified *CAS* (Computer Algebra System, *Maple*, *Mathematica*, or *Matlab* being popular examples). In either case texts and problem sets form an entity without gaps or jumps. And many problems can be solved by hand or with a computer or both ways. (For *software*, see the beginnings of *Part E* on Numeric Analysis and *Part G* on Probability and Statistics.)

More specifically, this new edition on the one hand gives more prominence to tasks the computer *cannot* do, notably, modeling and interpreting results. On the other hand, it includes *CAS projects*, *CAS problems*, and *CAS experiments*, which *do require* a computer and show its power in solving problems that are difficult or impossible to access otherwise. Here our goal is the combination of *intelligent* computer use with high-quality mathematics. This has resulted in a change from a formula-centered teaching and learning of engineering mathematics to a more quantitative, project-oriented, and visual approach. *CAS experiments* also exhibit the computer as an instrument for observations and experimentations that may become the beginnings of new research, for “proving” or disproving conjectures, or for formalizing empirical relationships that are often quite useful to the engineer as working guidelines. These changes will also help the student in discovering the *experimental aspect of modern applied mathematics*.

Some *routine and drill work* is retained as a necessity for keeping firm contact with the subject matter. In some of it the computer can (but must not) give the student a hand, but there are plenty of problems that are more suitable for pencil-and-paper work.

## Major Changes

**1. New Problem Sets.** Modern engineering mathematics is mostly *teamwork*. It usually combines analytic work in the process of modeling and the use of computer algebra and numerics in the process of solution, followed by critical evaluation of results. Our problems—some straightforward, some more challenging, some “thinking problems” not accessible by a *CAS*, some open-ended—reflect this modern situation with its increased emphasis on qualitative methods and applications, and the problem sets take care of this novel situation by including team projects, *CAS projects*, and writing projects. The latter will also help the student in writing general reports, as they are required in engineering work quite frequently.

**2. Computer Experiments**, using the computer as an instrument of “*experimental mathematics*” for exploration and research (see also above). These are mostly open-ended

experiments, demonstrating the use of computers in experimentally finding results, which may be provable afterward or may be valuable heuristic qualitative guidelines to the engineer, in particular in complicated problems.

**3. More on modeling and selecting methods**, tasks that usually cannot be automated.

**4. Student Solutions Manual and Study Guide enlarged**, upon explicit requests of the users. This Manual contains worked-out solutions to carefully selected odd-numbered problems (to which App. 1 gives only the final answers) as well as general comments and hints on studying the text and working further problems, including explanations on the significance and character of concepts and methods in the various sections of the book.

## Further Changes, New Features

- Electric circuits moved entirely to Chap. 2, to avoid duplication and repetition
- Second-order ODEs and Higher Order ODEs placed into two separate chapters (2 and 3)
- In Chap. 2, applications presented before variation of parameters
- Series solutions somewhat shortened, without changing the order of sections
- Material on Laplace transforms brought into a better logical order: partial fractions used earlier in a more practical approach, unit step and Dirac's delta put into separate subsequent sections, differentiation and integration of transforms (not of functions!) moved to a later section in favor of practically more important topics
- Second- and third-order determinants made into a separate section for reference throughout the book
- Complex matrices made optional
- Three sections on curves and their application in mechanics combined in a single section
- First two sections on Fourier series combined to provide a better, more direct start
- Discrete and Fast Fourier Transforms included
- Conformal mapping presented in a separate chapter and enlarged
- Numeric analysis updated
- Backward Euler method included
- Stiffness of ODEs and systems discussed
- List of software (in Part E) updated; another list for statistics software added (in Part G)
- References updated, now including about 75 books published or reprinted after 1990

## Suggestions for Courses: A Four-Semester Sequence

The material, when taken in sequence, is suitable for four consecutive semester courses, meeting 3–4 hours a week:

1st Semester.	<i>ODEs</i> (Chaps. 1–5 or 6)
2nd Semester.	<i>Linear Algebra. Vector Analysis</i> (Chaps. 7–10)
3rd Semester.	<i>Complex Analysis</i> (Chaps. 13–18)
4th Semester.	<i>Numeric Methods</i> (Chaps. 19–21)



## Suggestions for Independent One-Semester Courses

The book is also suitable for various independent one-semester courses meeting 3 hours a week. For instance:

- Introduction to ODEs (Chaps. 1–2, Sec. 21.1)
- Laplace Transforms (Chap. 6)
- Matrices and Linear Systems (Chaps. 7–8)
- Vector Algebra and Calculus (Chaps. 9–10)
- Fourier Series and PDEs (Chaps. 11–12, Secs. 21.4–21.7)
- Introduction to Complex Analysis (Chaps. 13–17)
- Numeric Analysis (Chaps. 19, 21)
- Numeric Linear Algebra (Chap. 20)
- Optimization (Chaps. 22–23)
- Graphs and Combinatorial Optimization (Chap. 23)
- Probability and Statistics (Chaps. 24–25)

## Acknowledgments

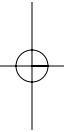
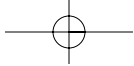
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*Suggestions of many readers worldwide were evaluated in preparing this edition. Further comments and suggestions for improving the book will be gratefully received.*

ERWIN KREYSZIG



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