國立中正大學九十九學年度學士班二年級轉學生招生考試試題

數學系、地球與環境科學系、物理學系
學系別：化學暨生物化學系、資訊工程學系　　　　科目：微積分
機械工程學系、通訊工程學系

第 1 節　　

PART I.　Fill in the blank (9 points each. No partial credits)

1. $\lim\limits_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\dots+\frac{1}{2n}\right)$ = ______.

2. If f (x) is a function satisfying $f'(x)=\frac{\sin x}{x}$, $f(\frac{\pi}{2})=a$, $f(\frac{7\pi}{3})=b$,

then $\int_{\pi/2}^{7\pi/3} f(x)\,dx$ = ________. (express your answer in terms of a and b).

3. If $\int_1^{\infty}\left(\frac{x}{ax^2+1}-\frac{2}{3x}\right)dx$ is a convergent improper integral, then a =_______.

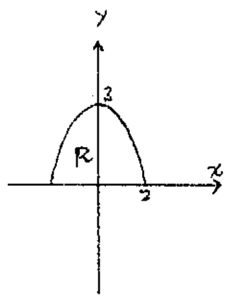
and the value of the improper integral is _________.

4. The interval of convergence of the power series $\sum\frac{4^n}{n+1}(x-2)^n$ is ______.

5. The line integral $\int_C F\cdot dr$ = _______, where $F=\frac{-yj+xi}{4x^2+y^2}$,

and C is the unit circle traced counterclockwise.

6. $\iint_R \sin(9x^2+4y^2)\,dA$ =_______, where R is part of the ellipse in the right figure.



7. $\lim\limits_{n\to\infty}\frac{1}{n}(1+\sqrt{2}+\sqrt[3]{3}+\dots+\sqrt[n]{n})$ = _________.

8. The area of the region that is inside both curves: the cardioid r = 1+cos θ and the circle r = 1 is _________.

PART II. Show your work to get full credits. (14 points each)

1. (a) Show that $\int_0^{\infty} xe^{-nx}dx=\frac{1}{n^2}$, n = 1, 2, 3….

(b) Use above result to show that $\int_0^{\infty}\frac{x}{e^x-1}dx=\sum_{n=1}^{\infty}\frac{1}{n^2}$

2. Let $y=h(t)=\frac{1}{1+9e^{-0.08t}}$ then y satisfies the differential equation $y'=f(y)$.

(a) Find the function $f(y)$.

(b)The function $y=h(t)$ has a unique inflection point in (0 , ∞), find this point.